Concordia University

Department of Computer Science & Software Engineering

COMP 335/4 **Theoretical Computer Science** Winter 2015

Midterm Exam February 19, 2015

Your name:..............................StudentID:...................

Your signature:...

Write all answers in this booklet. No points will be awarded for answers appearing elsewhere. Write your answers with pen, not pencil.

1. For each of the languages L_i, $i \in \{1, 2\}$, give a regular expression R_i, such that $L(R_i) = L_i$.

   (a) $L_1 = \{w \in \{a, b\}^* : \text{ length of } w \text{ is odd }\}$

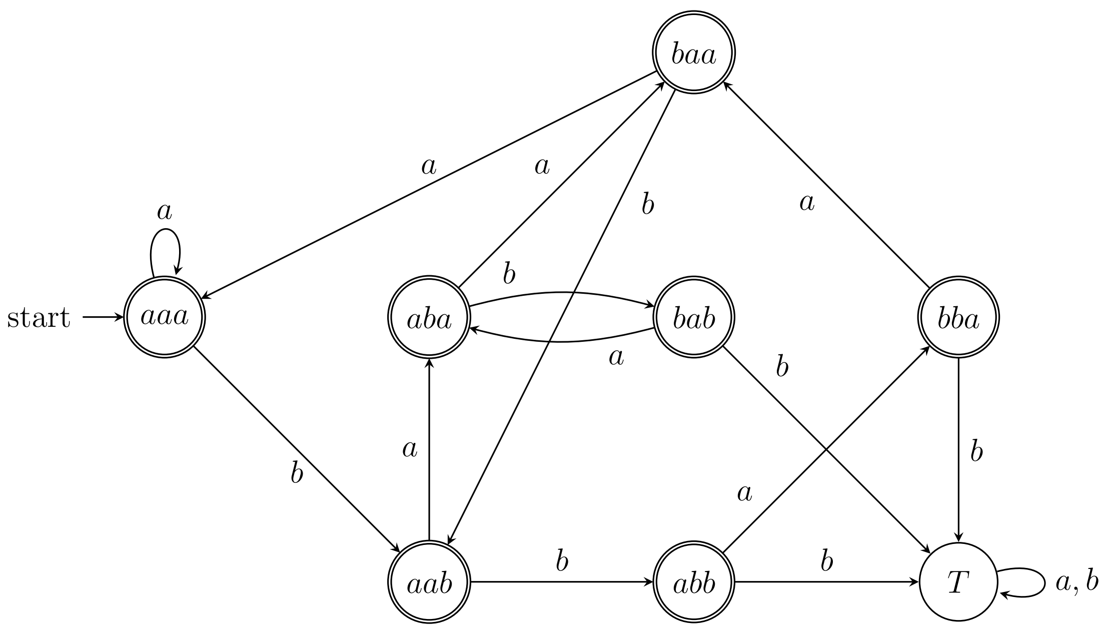
   Solution: $(aa + bb + ab + ba)^*(a + b)$

   (b) $L_2 = \{w \in \{a, b\}^* : w \text{ has exactly one occurrence of } aab\}$

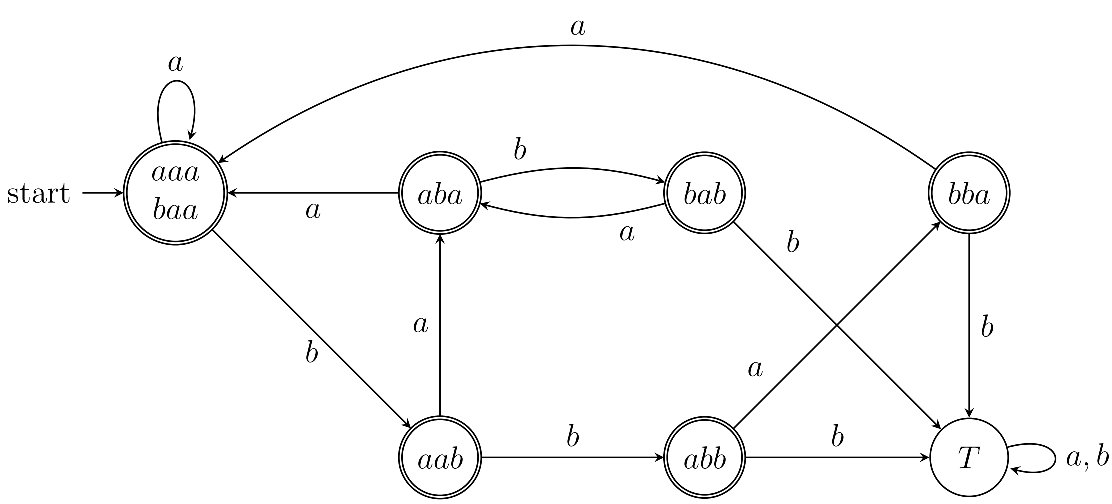
   Solution: $(b + ab)^*a^*aab(b + ab)^*a^*$

2. Let the alphabet $\Sigma = \{a, b\}$. Construct a DFA that accepts language consisting of all strings where every substring[1] of four symbols has at most two b's. For example, *bbaab* and *baabba* are in the language, while for example *abbab* is not in the language. Give your DFA as a transition diagram.

   Solution: The automaton has to remember the last three symbols read, so we need at least $2^3 = 8$ states. Each state is labelled by the three last symbols read. Note that *bbb* can be identified with the trap-state.



   This can be minimized to

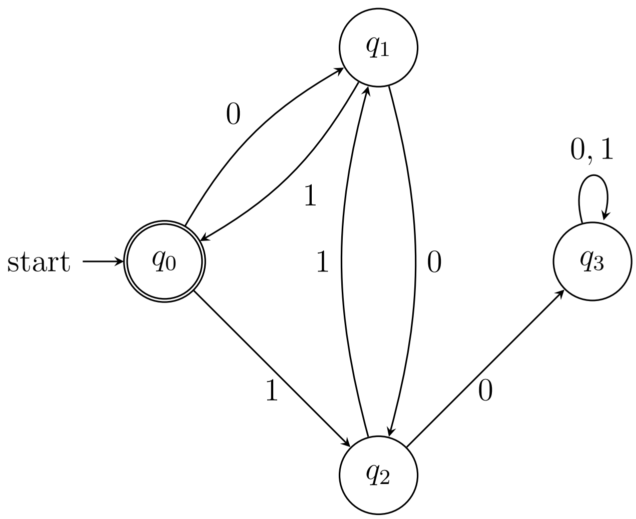


[1]A string v is a substring of a string w, if there are strings x and y (possibly ϵ), such that $w = xvy$.

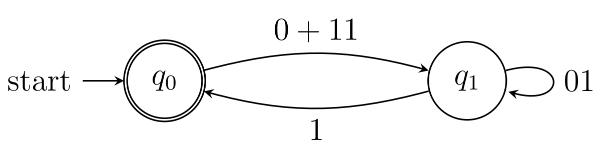
3. Use the state-elimination technique to find a regular expression for the DFA A below.

A	0	1
$\rightarrow * q_0$	q_1	q_2
q_1	q_2	q_0
q_2	q_3	q_1
q_3	q_3	q_3

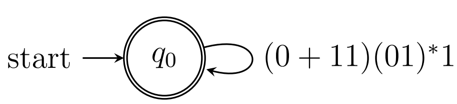
Solution:



Eliminating states q_2 and q_3 (trap state), we get:



Eliminating state q_1, we get:



Finally, $r = ((0 + 11)(01)^*1)^*$.

4. Let L be the language of those strings over $\{0, 1\}$ where the number of 0's differ from the number of 1's by at most 5. Prove that L is not regular.

Solution:

Suppose L is regular. Then there exists a DFA A with $L(A) = L$. Suppose A has n states. Choose $w = 0^n 1^n$. Then $w \in L$. Since $|w| \geq n$, the pumping lemma tells us that w can be decomposed as xyz, where $|xy| \leq n$, $|y| \geq 1$. Therefore y must consist of one or more zeroes. The pumping lemma also tells us that xy^7z is in $L(A)$. But xy^7z has at least 6 more zeroes than ones (since $|y| \geq 1$), and is not in L. Therefore it cannot be that $L = L(A)$, and the assumption that L is regular is wrong.