Assignment 4

Due date: April 14, 2015, by 11:59 p.m. EDT

1. (a) Construct a Turing machine for the language

$$\{xay \ : \ x, y \in \{a, b\}^*, |x| = |y|\}.$$

(b) Construct a Turing machine for the language

$$\{a^n b^{2n} c^{3n} : n \geq 1\}.$$

(c) Construct a Turing machine that creates a copy of its input string to the right of the input with a blank separating the copy from the original.

2. Consider a nondeterministic TM whose tape is infinite in both directions. At some time, the tape is completely blank, except for one cell, which holds the symbol \$. The head is currently at some blank cell, and the state is q.

(a) Write transitions that will enable the NTM to enter state p upon reading \$.

(b) Suppose the TM were deterministic instead. How would you enable it to find the \$ and enter state p?

3. Give a complete encoding of the TM

$$A = (\{q_1, q_2, q_3\}, \{a_1, a_2\}, \delta, B, \{q_3\}),$$

where the transition function δ is given by $\delta(q_1, a_1) = (q_1, a_1, R), \delta(q_1, a_2) = (q_3, a_1, L), \delta(q_3, a_1) = (q_2, a_2, L)$.

4. For each of the languages

$$L_i = \{w_i : w_i \notin L(M_{2i})\}$$

and

$$L_{2i} = \{w_i : w_{2i} \notin L(M_i)\}$$

show that the language is not accepted by any TM, using a diagonalization-type argument, as was done for the language L_d.

5. Prove the following statement:

A language L is recursive if and only if both L and $\bar{L}$ are recursively enumerable.

6. Show that the language of codes for TM's M that, when started with a blank tape, eventually write a "1" somewhere on the tape is undecidable.

7. For each of the following instances (A, B) of Post's Correspondence Problem, determine if it has a solution or not. If you think (A, B) has a solution, give one, and if you think (A, B) does not have a solution, provide reasoning to justify your claim.

(a)

	List A	List B
i	w_i	x_i
1	11	111
2	100	001
3	111	11

(b)

	List A	List B
i	w_i	x_i
1	00	0
2	001	11
3	1000	011