Concordia University
Introduction to Theoretical Computer Science
Winter 2015
Solution to Assignment 4.

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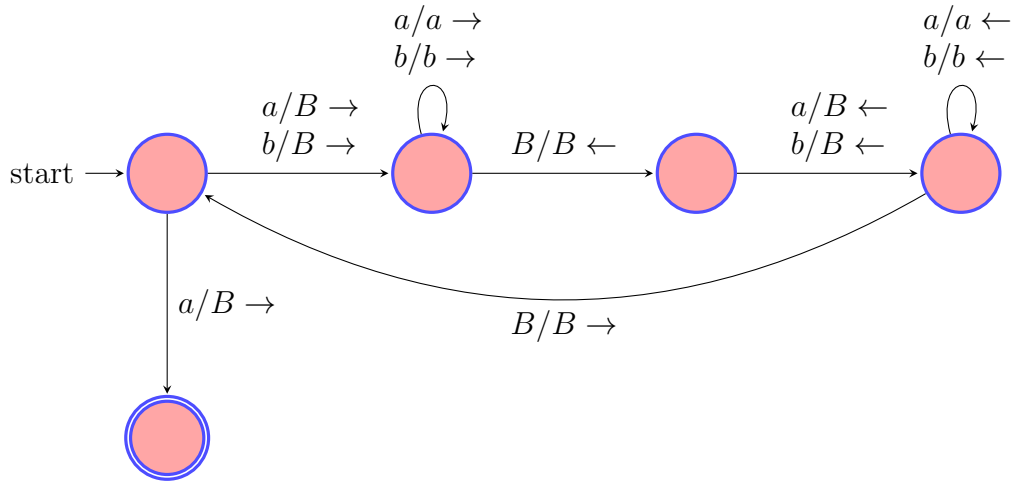
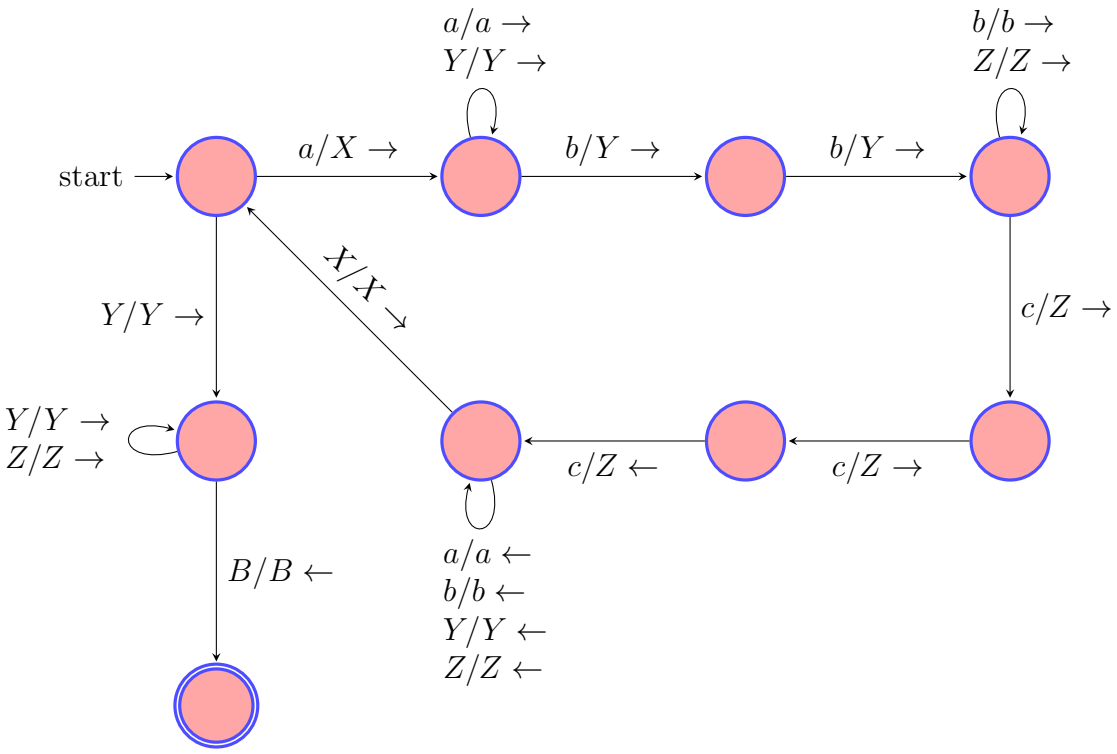
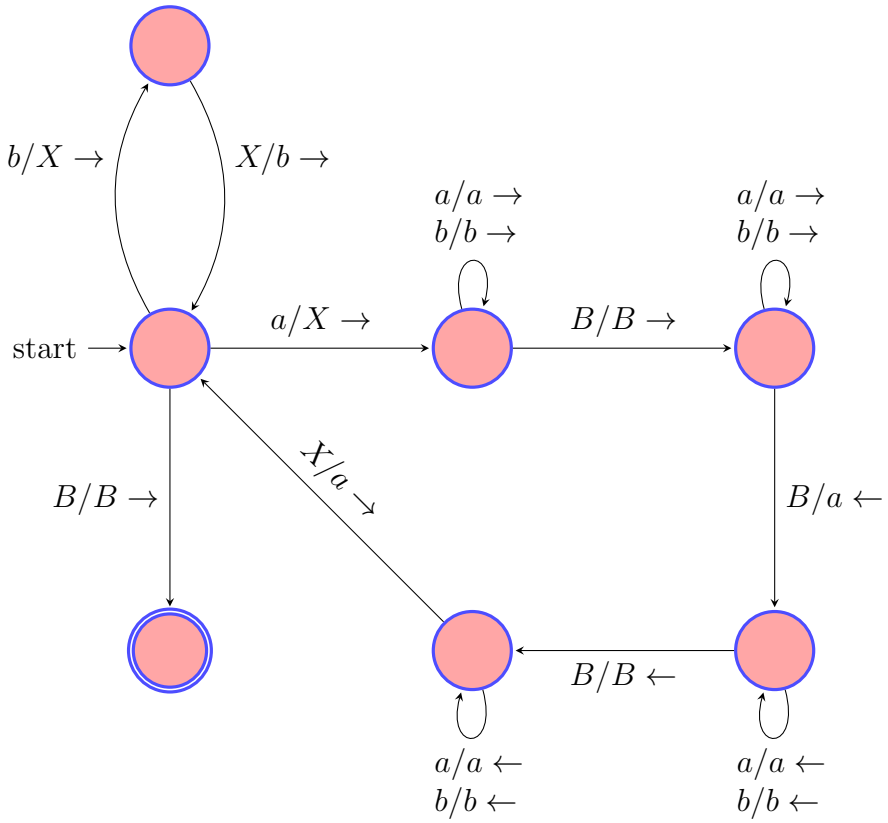
1. (a) NTM:



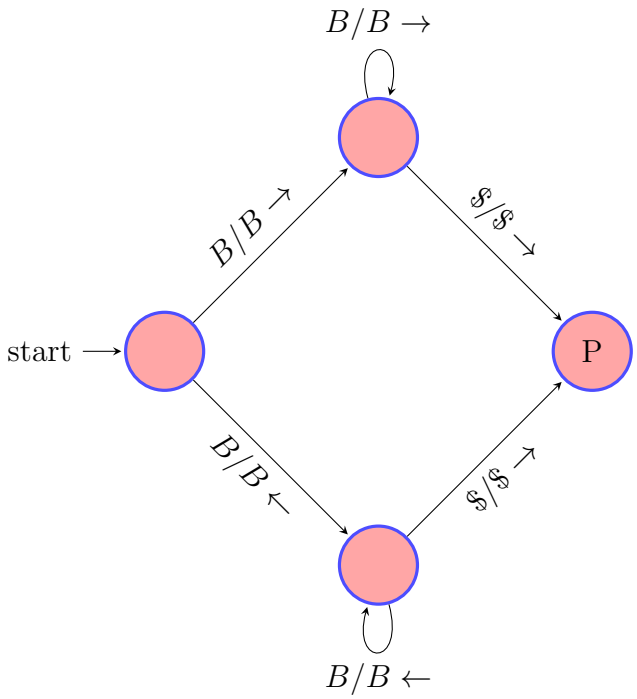
(b) Turing machine:



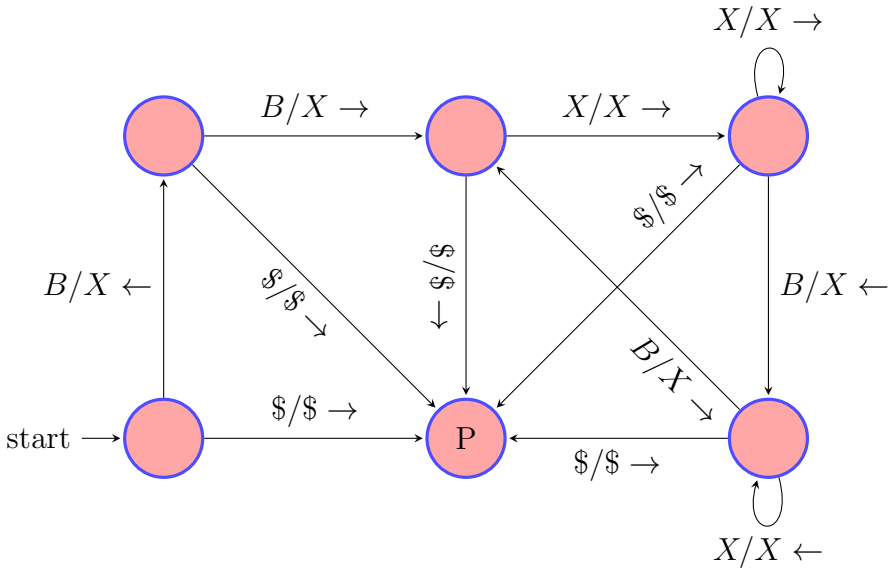
(c) Storage in state:



2. (a)


$B/B \rightarrow$
$B/B \rightarrow$
$\$/\$ \rightarrow$
start
P
$B/B \leftarrow$
$\$/\$ \rightarrow$
$B/B \leftarrow$

(b)


$X/X \rightarrow$
$B/X \rightarrow$
$X/X \rightarrow$
$\$/\$ \rightarrow$
$B/X \leftarrow$
$\$/\$ \rightarrow$
$\$/\$ \downarrow$
$B/X \leftarrow$
$B/X \rightarrow$
start
$\$/\$ \rightarrow$
P
$\$/\$ \rightarrow$
$X/X \leftarrow$

3. The complete encoding of the TM is as follows:

$q_1 \mapsto 0$

$q_2 \mapsto 00$

$q_3 \mapsto 000$

$a_1 \mapsto 0$

$a_2 \mapsto 00$

$B \mapsto 000$

$L \mapsto 0$

$R \mapsto 00$

$\delta(q_1, a_1) = (q_1, a_1, R) \mapsto \overbrace{0101010100}^{c_1}$

$\delta(q_1, a_2) = (q_3, a_1, L) \mapsto \overbrace{010010001010}^{c_2}$

$\delta(q_3, a_1) = (q_2, a_2, L) \mapsto \overbrace{0001010010010}^{c_3}$

TM: $c_1 \,||\, c_2 \,||\, c_3$

$\quad x \;||\, y \;||\, z \,\#$

4. (a) $L_i = \{w_i : w_i \notin L(M_{2i})\}$

		w_i			
		1	2	3	4
	1				
	2	①			
	3	0			
	4		⓪		
M_i	5		1		
	6			①	
	7			0	
	8				⓪
	9				1

Suppose L_i is $L(M_j)$ for some $j = 1, 2, \ldots$

Case 1: j even: $\begin{cases} w_{i/2} \in L(M_i) \Rightarrow w_{i/2} \notin L_i \\ w_{i/2} \notin L(M_i) \Rightarrow w_{i/2} \in L_i \end{cases}$

Case 2: j odd: not possible, it always ends in 0.

(b) $L_2 i = \{w_i : w_2 i \notin L(M_i)\}$

		w_i							
		1	2	3	4	5	6	7	8
	1		◯						
M_i	2				◯				
	3						◯		
	4								◯

Suppose $L_{2i} = L(M_j)$ for some $j = 1, 2, \ldots$

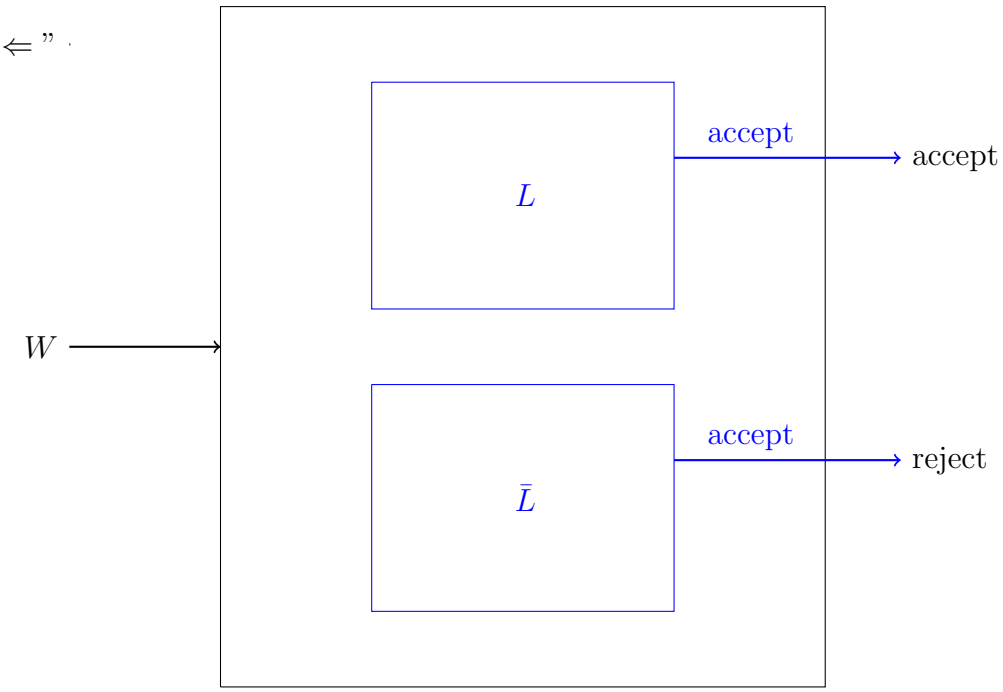
$$\begin{aligned} w_j \in L(M_j) &\Rightarrow w_{2j} \notin L(M_j) \\ &\Rightarrow w_j \notin L_{2i} \end{aligned}$$

$$\begin{aligned} w_j \notin L(M_j) &\Rightarrow w_{2j} \in L(M_j) \\ &\Rightarrow w_j \in L_{2i} \end{aligned}$$

5. ” $\Rightarrow$ ” L is REC $\Rightarrow L$ is RE

” $\Rightarrow$ ” $\bar{L}$ is REC $\Rightarrow \bar{L}$ is RE

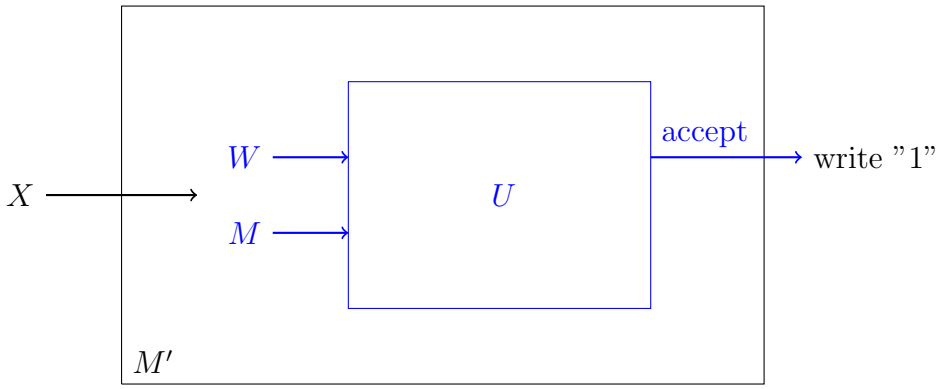
” $\Leftarrow$ ” .



6. Call the language L_1.

   Now Suppose that L_1 is decidable.

   Reduction from L_U:



M' writes "1" $\Leftrightarrow (M', w) \in L_U$; $[w \in L(M)]$

7. (a)

$$\overbrace{11}^{w_1}\overbrace{100}^{w_2}\overbrace{111}^{w_3}$$

$$\underbrace{111}_{x_1}\underbrace{001}_{x_3}\underbrace{11}_{x_3}$$

(b) No solution:

$$\left.\begin{array}{l} |w_1| > |x_1| \\ |w_2| > |x_2| \\ |w_3| > |x_3| \end{array}\right\} \quad (*)$$

Now suppose: $i_1, i_2, \ldots, i_n$ is a solution;

$(*) \Rightarrow |w_{i_1} w_{i_2} \ldots w_{i_n}| > |x_{i_1} x_{i_2} \ldots x_{i_n}|$