CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE & SOFTWARE ENGINEERING

COMP 335/4 Theoretical Computer Science Winter 2015

Assignment 3

Due date: March 27, 2015, by 23:59 p.m. EDT

1. In each case, what language is generated by CFG's below. Justify your claim (prove it!)
   (a) $S \to aSa|bSb|aAb|bAa,\ A \to aAa|bAb|a|b|\epsilon$
   (b) $S \to aS|bS|a$
   (c) $S \to SS|bS|a$
   (d) $S \to SaS|b\ S \to aT|bT|\epsilon,\ T \to aS|bS.$

2. Find a CFG for each of the languages below.
   (a) $L = \{a^n b^m : n \neq m - 1\}$
   (b) $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$
   (c) $L = \{w \in \{a,b\}^* : n_a(w) \neq n_b(w)\}$
   (d) $\overline{L}$, where $L = \{w \in \{a,b\}^* : w = a^n b^n,\ n \geq 0\}$

3. In each case below, show that the grammar is ambiguous, and find an equivalent unambiguous grammar.
   (a) $S \to SS|ab|a$
   (b) $S \to ABA,\ A \to aA|\epsilon,\ B \to bB|\epsilon$
   (c) $S \to aSb|aaSb|\epsilon$

4. Design a PDA to accept each of the following languages. You may design your PDA to accept either by final state or empty stack, whichever is more convenient.
   (a) The set of strings over $\{0,1\}$ such that no prefix has more 1's than 0's.
   (b) The set of strings with twice as many 0's as 1's.
   (c) The set of strings over $\{a,b\}$ that are *not* of the form ww, that is, not equal to any string repeated.

5. Construct a PDA corresponding to the context-free grammar
   $S \to SS \mid \{SX \mid [SY \mid \epsilon$
   $X \to \}$
   $Y \to]$
   Note that $\{, [,]$, and $]$ are terminals.

6. Consider the PDA $P = \{\{q_0, q_1, q_2\}, \{a\}, \{\clubsuit, Z_0\}, \delta, q_0, Z_0, \{q_2\}\}$, where $\delta(q_0, a, Z_0) = \{(q_1, \clubsuit Z_0)\}$, $\delta(q_1, a, \clubsuit) = \{(q_0, \epsilon)\}$, and $\delta(q_0, \epsilon, Z_0) = \{(q_2, \epsilon)\}$.
   Construct a CFG (using the method in the text) corresponding to P.

7. Use the Pumping Lemma for CFL's to show that none of the following languages are context-free.
   (a) $L_1 = \{ww\ :\ w \in \{a,b\}^*\}$
   (b) $L_2 = \{a^n b^k\ :\ 0 \leq n \leq k^2\}$
   (c) $L_3 = \{a^n b^m c^k\ :\ 0 \leq n < m,\ n \leq k \leq m\}$

8. Convert the following grammar into Chomsky normal form
   $S \to aA|aBB$
   $A \to aaA|\epsilon$
   $B \to bB|bbC$
   $C \to C|B$

9. (a) Show that the language
   $$L = \{a^n b^n : a, b \in \{a,b\},\ n \text{ is not a multiple of } 5\}$$
   is context-free.
   (b) Let $L = \{a^n b^n : n \geq 0\}$, and $M = \{a^{2m} b^{2p} : m \geq 0, p \geq 0\}$. Construct a PDA for L and a DFA[1] for M. Then use the Cartesian construction to obtain a PDA for $L \cap M$.

[1] Leave out the trap state