Concordia University
Introduction to Theoretical Computer Science
Winter 2015

Solution to Assignment 2 - Part II

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5. If L is regular, it is accepted by some DFA, say $A = (Q, \Sigma, \delta, s_0, F)$. We will construct an ϵ-NFA, such that $L(B) = third(L(A))$. Here you need four copies of A. Formally,

$$B = (Q \times \{1, 2, 3, 4\}, \Sigma, \rho, F \times \{2, 3, 4\}),$$

where $\rho =$

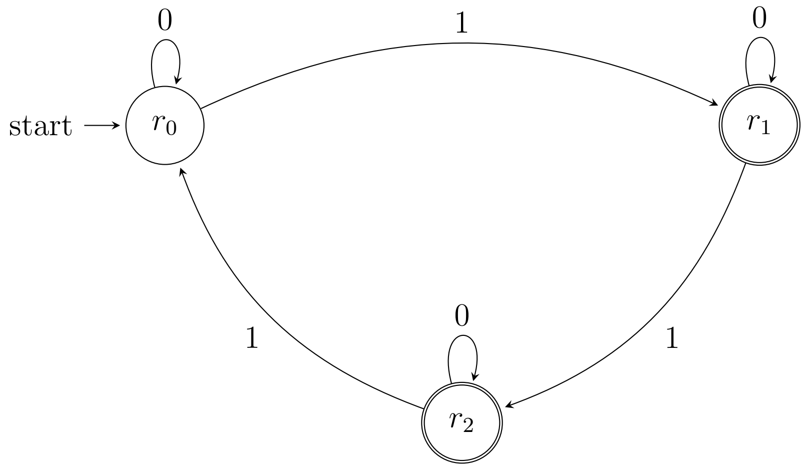
$\{(\langle p, 1\rangle, \epsilon, \langle q, 2\rangle) : (p, a, q) \in \delta, \text{ for some } a \in \Sigma\} \cup$

$\{(\langle p, 2\rangle, \epsilon, \langle q, 3\rangle) : (p, a, q) \in \delta, \text{ for some } a \in \Sigma\} \cup$

$\{(\langle p, 3\rangle, \epsilon, \langle q, 4\rangle) : (p, a, q) \in \delta\} \cup$

$\{(\langle p, 4\rangle, \epsilon, \langle p, 1\rangle) : p \in Q\}$

6. The following DFA describes the language L.

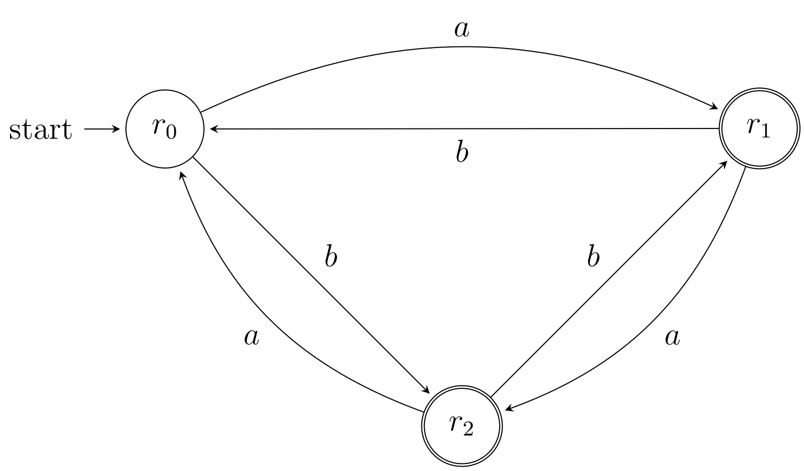


Call this DFA A. Clearly $L(A) = L$. More precisely, $A = (\{r_0, r_1, r_2\}, \{0, 1\}, \delta, \{r_1, r_2\})$. Construct $B = (\{r_0, r_1, r_2\}, \{0, 1\}, \gamma, \{r_1, r_2\})$, where

$$\gamma(q, a) = \hat{\delta}(q, h(a)),$$

for all $q \in \{r_1, r_2, r_3\}$ and all $a \in \{0, 1\}$.

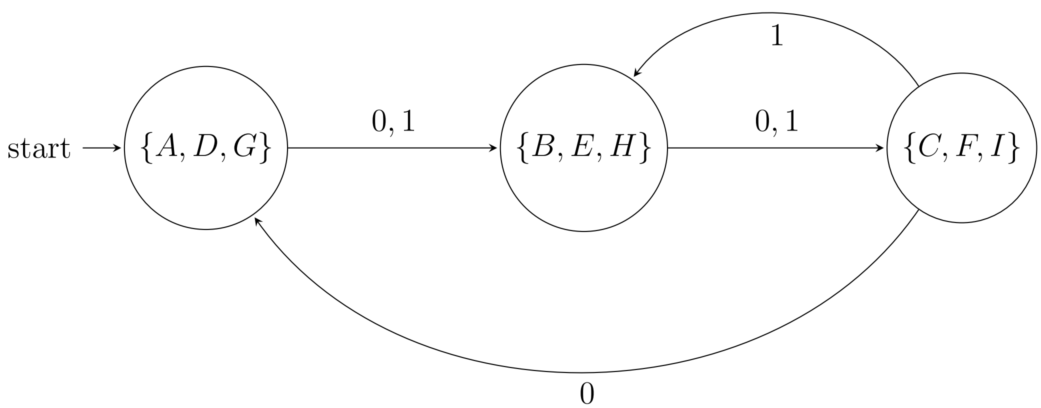
Then $L(B) = h^{-1}(L(A)) = h^{-1}(L)$. The transition diagram for B is shown below.



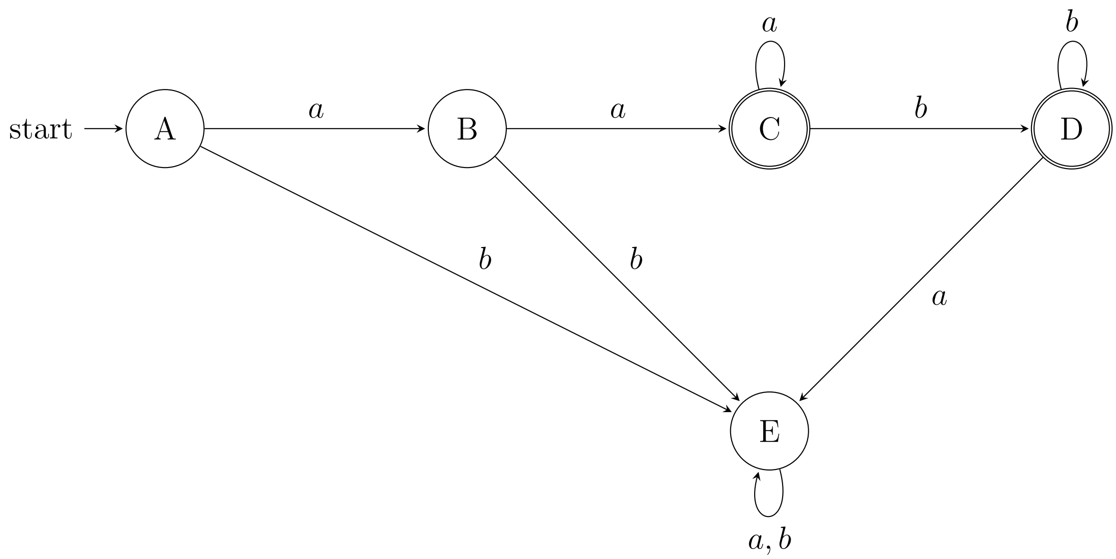
7. Table of distinguishabilities:

	A	B	C	D	E	F	G	H
B	×							
C	×	×						
D	○	×	×					
E	×	○	×	×				
F	×	×	○	×	×			
G	○	×	×	○	×	×		
H	×	○	×	×	○	×	×	
I	×	×	○	×	×	○	×	×

- Using the table we can construct the minimum state equivalent DFA.



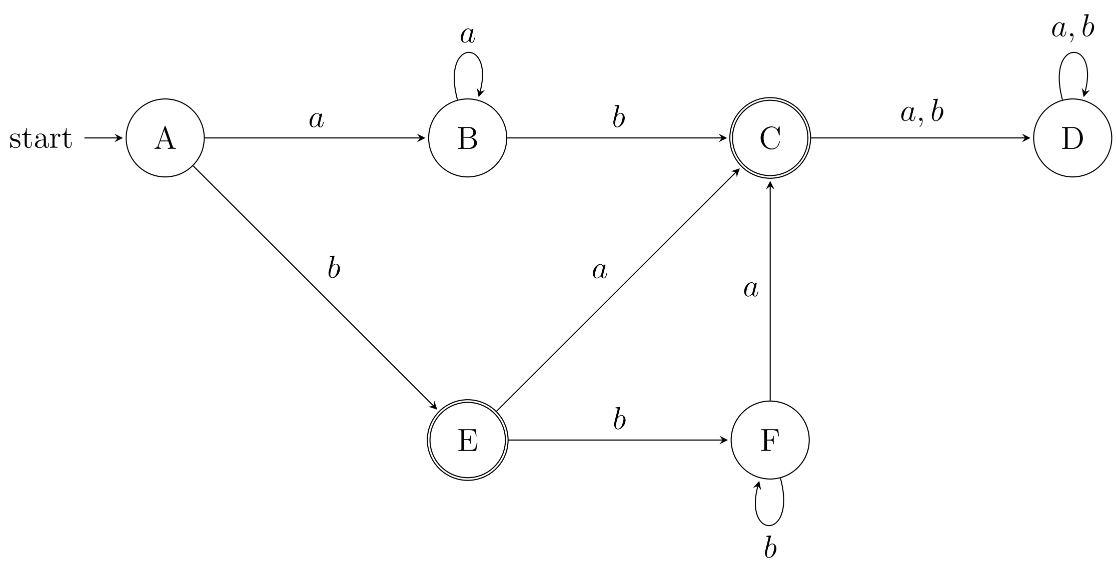
8. (a) $\{a^n b^m : n \geq 2, m \geq 1\}$



To show that this automaton is minimal, we compute its table of distinguishabilities:

B	×			
C	×	×		
D	×	×	×	
E	×	×	×	×
	A	B	C	D

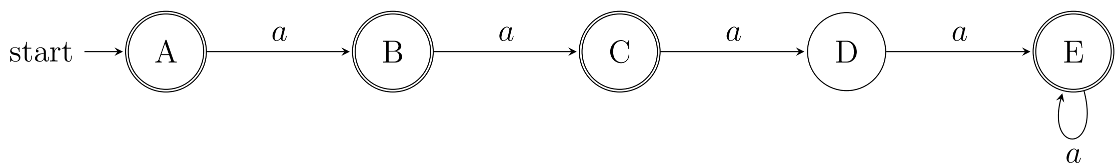
(b) $\{a^n b : n \geq 0\} \cup \{ab^n : n \geq 1\}$



To show that this automaton is minimal, we compute its table of distinguishabilities:

	A	B	C	D	E
B	×				
C	×	×			
D	×	×	×		
E	×	×	×	×	
F	×	×	×	×	×

(c) $\{a^n b : n \geq 0\} \cup \{ab^n : n \geq 1\}$



To show that this automaton is minimal, we compute its table of distinguishabilities:

	A	B	C	D
B	×			
C	×	×		
D	×	×	×	
E	×	×	×	×