CONCORDIA UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE & SOFTWARE ENGINEERING

COMP 335/4 — Theoretical Computer Science — Winter 2015

Assignment 2

Due date: Part I : February 16, 2015, Part II: March 2, 2015.

Part I

1. Give a regular expression for each of the languages below.

   (a) $\{aa, ab, ba, bb\} \setminus \{aa, bb\}$.

   (b) $\{a^k b^m c^n \ : \ k+m+n \text{ is odd}\}$.

   (c) $\{w \in \{a,b,c\}^* \ : \text{ no symbol occurs twice in succession in } w\}$.

   (d) $\{w \in \{0,1\}^* \ : \ 00 \text{ occurs at most twice in } w\}$.
   Note: 00 occurs twice in 000

2. Use the state-elimination technique to find a regular expression for

   (a) the DFA given by the following transition table:

	0	1
$\rightarrow \star\, q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

   (b) the DFA given by the following transition table:

	a	b	c
$\rightarrow q_1$	q_6	q_2	q_4
q_2	q_3	q_6	q_6
q_3	q_4	q_5	q_6
q_4	q_2	q_6	q_5
$\star\, q_5$	q_6	q_6	q_6
q_6	q_6	q_6	q_6

3. Convert the following regular expressions to ϵ-NFA's.

   (a) $(\mathbf{000})^*(\epsilon + \mathbf{011} + \mathbf{001})(\mathbf{111})^*$

   (b) $(\mathbf{0} + \mathbf{1})^*(\mathbf{001} + \mathbf{010} + \mathbf{100})^*(\mathbf{0} + \mathbf{1})^*$

   (c) $(\mathbf{01} + \mathbf{10})^* + (\mathbf{00} + \mathbf{11})^* + (\mathbf{1} + \mathbf{10} + \mathbf{100})^*$

4. Apply the Pumping Lemma to prove that the following languages are not regular.

   (a) $\{a^k b^n \ : \ n = 2^k\}$

   (b) $\{a^n b^m a^k : n = m \text{ or } m \neq k\}$

   (c) $\{a^n : n \text{ is a product of two primes}\}$

Part II

5. For a string $w = a_1a_2a_3a_4a_5a_6a_7\ldots$, define $third(w) = a_3a_6a_9\ldots$. Then, for a language L, define $third(L) = \{third(w) : w \in L\}$. Show that if L is regular, then $third(L)$ is also regular.

   Hint: Construct an ϵ-NFA from the DFA for L.

6. Let h be the homomorphism $h : \{a,b\} \rightarrow \{0,1\}^*$ given by $h(a) = 01$, $h(b) = 011$, and define $L = \{w \in \{0,1\}^* \ : \ n_1(w) \not\equiv 0 \ (\textbf{mod}\ 3)\}$. Construct a DFA for $h^{-1}(L)$.

7. Draw the table of distinguishabilities for the DFA below (run the TF algorithm), and then construct the minimum state equivalent DFA.

	0	1
$\rightarrow A$	B	E
B	C	F
$*\, C$	D	H
D	E	H
E	F	I
$*\, F$	G	B
G	H	B
H	I	C
$*\, I$	A	E

8. Find minimal DFA's for the following languages. In each case prove (!) that your DFA is minimal.

   (a) $\{a^n b^m : n \geq 2, m \geq 1\}$

   (b) $\{a^n b : n \geq 0\} \cup \{b^n a : n \geq 1\}$

   (c) $\{a^n : n \geq 0, n \neq 3\}$