Concordia University
Introduction to Theoretical Computer Science
Winter 2015

Solution to Assignment 2 - Part I

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1. (a)

$$e = ab + ba$$

(b)

$$e = a(aa)^*(bb)^*(cc)^* + (aa)^*b(bb)^*(cc)^* + (aa)^*(bb)^*c(cc)^* + a(aa)^*b(bb)^*c(cc)^*$$

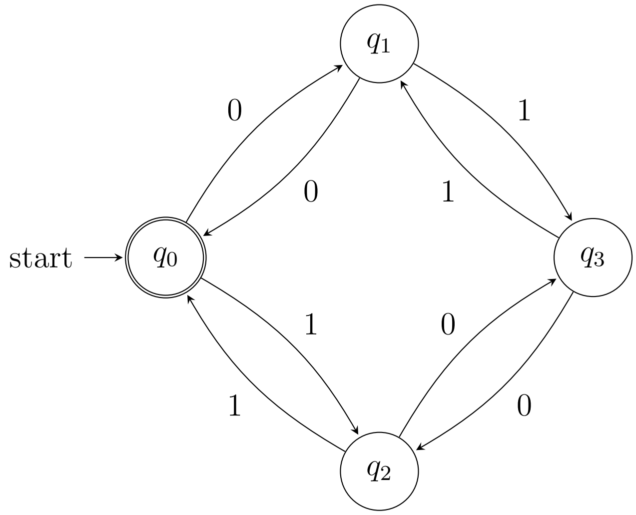
(c)

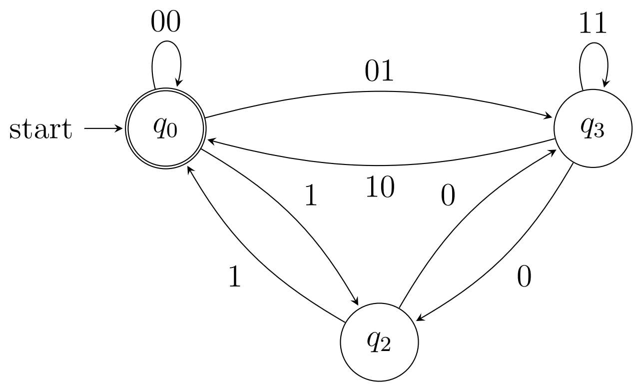
$$\begin{aligned} e =& (ab + ac + abc + acb + abcb + acbc)^*(\epsilon + a) + \\ & (ba + bc + bac + bca + bcac + baca)^*(\epsilon + b) + \\ & (ca + cb + cba + cab + cbab + caba)^*(\epsilon + c) + \end{aligned}$$

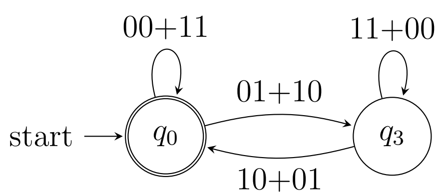
(d)

$$e = 1^*(011^*)^*(00 + \epsilon)((11^*0)^*0 + \epsilon)(11^*0)^*$$

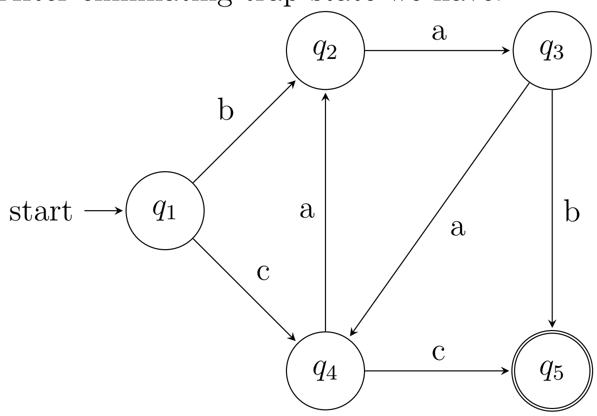
2. (a) Eliminating states q_1 and q_2



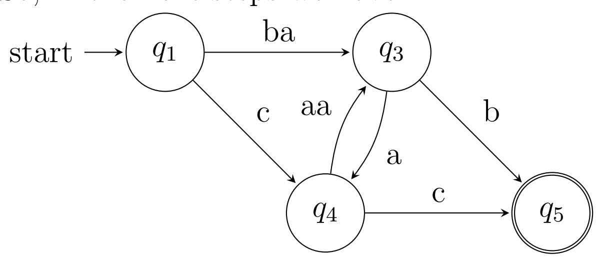


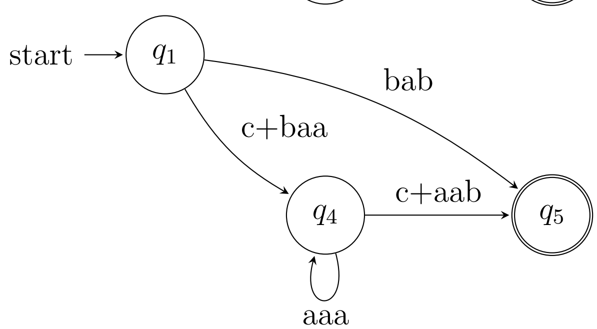


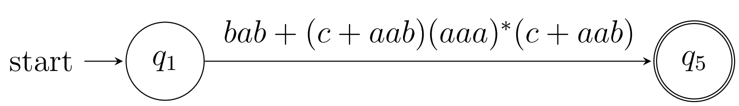
(b) After eliminating trap state we have:



So, in the next steps we have:



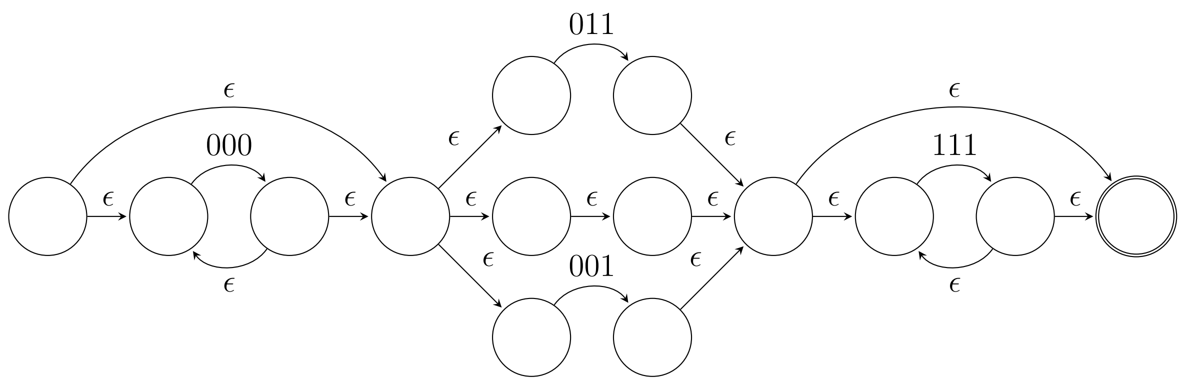




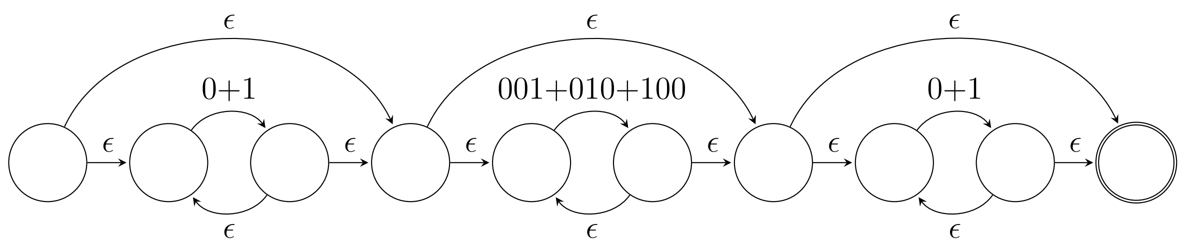
As a result, we have: $r.e. = bab + (c + aab)(aaa)^*(c + aab)$

3. ** **Notice:** Answers are simplified **

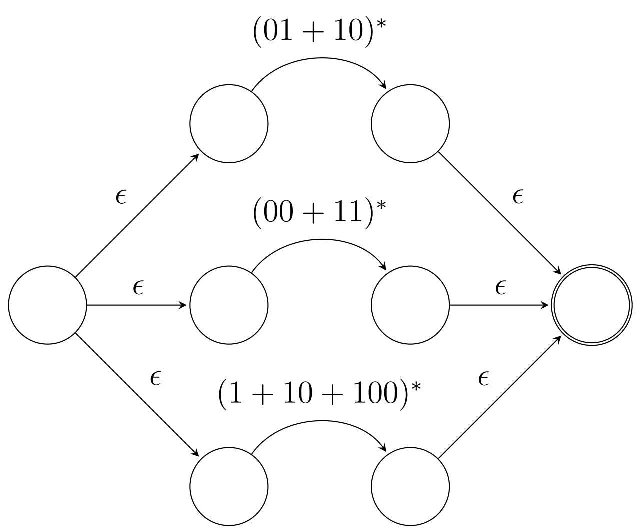
(a) $(000)^*(\epsilon + 011 + 001)(111)^*$



(b) $(0 + 1)^*(001 + 010 + 100)^*(0 + 1)^*$



(c) $(01 + 10)^* + (00 + 11)^* + (1 + 10 + 100)^*$



4. (a) Let G be a finite state automaton corresponding to the language $L = \{a^k b^{2^k}\}$, and let $|G| = m$. While $a^m b^{2^m}$ is in L, so, G accepts it. Using pumping lemma, there are some x, y and z such that $|xy| = m$, $|y| > 0$, $xyz = a^m b^{2^m}$, and $xy = a^m$. By pumping lemma xy^*z is also in $L(G)$. While $|y| = p > 0$, and $xz = a^{m-p}b^{2^m}$ is in $L(G)$, however it is not in L, which is a contradiction with assumption that G is corresponding to L. Finally there is no such a G which means L is not regular.

(b) Let G be a finite state automaton corresponding to the language $L = \{a^n b^m c^k \ : \ n = m \, \text{or} \, m \neq k\}$, and let $|G| = m$. While $a^m b^m a^m$ is in L, so, G accepts it. Using pumping lemma, there are some x, y and z such that $|xy| = m$, $|y| = p > 0$ and $xyz = a^m b^m a^m$, and $xy = a^m$ such that xy^*z is also in $L(G)$. While $|y| = p > 0$, and $xz = a^{m-p}b^m a^m$ is in $L(G)$, however it is not in L, which is a contradiction with assumption that G is corresponding to L. Finally there is no such a G so L is not regular.

(c) Let G be a finite state automaton automaton corresponding to the language $L = \{a^{rs} \ : \ r, s \, \text{are prime}\}$, and let $|G| = m$. Moreover let r be prime number bigger than m, and s be any prime number. While a^{rs} is in L, so, G accepts it. Using pumping lemma, there are some x, y and z such that $|xy| = m$, $|y| = p > 0$ and $xyz = a^{rs} = a^{(m+(r-m))s} = a^{ms}a^{(r-m)s} = a^m a^{m(s-1)} a^{(r-m)s}$, and $xy = a^m$ such that xy^*z is also in $L(G)$. While $|y| = p > 0$, and $xy^{rs+1}z = a^{m-p}a^{p(rs+1)}a^{m(s-1)}a^{(r-m)s} = a^{m-p+p}a^{p(rs)}a^{m(s-1)}a^{(r-m)s} = a^m a^{m(s-1)}a^{(r-m)s}a^{p(rs)} = a^{rs}a^{p(rs)} = a^{(p+1)(rs)}$ is in $L(G)$, since $(p+1)rs$ is not a product of two prime numbers so $a^{(p+1)(rs)}$ is not in L, which is a contradiction with assumption that G is corresponding to L. Finally there is no such a G that is L is not regular.