Concordia University

Department of Computer Science & Software Engineering

COMP 335/4 — Theoretical Computer Science — Winter 2015

Assignment 1

Due date: Tuesday February 3, 2015, by 23:59 p.m. EST

1. The *reversal* of a string w, denoted by w^R, is the string "spelled backwards". For example $(cat)^R = tac$. Reversal is defined inductively as follows:

   $$\epsilon^R = \epsilon, (wa)^R = a(w^R).$$

   (Here a is a symbol in the alphabet Σ, and w is a string in Σ^*.)

   Let $u, v \in \Sigma^*$. Prove that

   $$(uv)^R = v^R u^R.$$

   Hint: Use induction on $|v|$.

2. Let $\Sigma = \{a, b\}$. For each of the languages below, give an example of a string in the language, and a string not in the language.

   (a) $\{w \in \Sigma^* : w = uu^R u, \text{ for some } u \in \Sigma^2\}$

   (b) $\{w \in \Sigma^* : ww = www\}$

   (c) $\{w \in \Sigma^* : uvw = wvu, \text{ for some } u, v \in \Sigma^*\}$.

   (d) $\{w \in \Sigma^* : www = uu, \text{ for some } u \in \Sigma^*\}$.

3. Construct a DFA for each of the following languages.

   (a) $\{w \in \{a, b\}^* : bb \text{ appears at most once as a substring of } w\}$

   (b) $\{w \in \{a, b\}^* : bab \text{ is not a substring of } w\}$

   (c) The set of strings that either begin or end (or both) with ab.

   (d) $\{w \in \{a, b\}^* : w \text{ contains an odd number of } a's \text{ and ends in at least two } b's\}$

   Give your DFA's as transition diagrams.

4. Let $L = \{w \in \{0,1\}^* : w \text{ has an odd no. of 1's }\}$, and let A be the DFA with tabular representation:

A	0	1
$\rightarrow p$	p	q
$\star q$	q	p

   Prove that $L = L(A)$. *Hint:* Do the $L(A) \subseteq L$ part of the proof by induction on the the length of the string processed by A. You need a mutual induction with a claim for state p and a claim for state q.

5. Construct an NFA for each of the following languages.

   (a) The set of strings over $\{0, 1, \ldots, 9\}$, such that the final digit has not appeared before

   (b) The set of strings over $\{0, 1\}$, such that there are two 0's separated by a number of positions that is a multiple of 4. Note that 0 is an allowable multiple of 4.

6. Let $\Sigma = \{a, b\}$.

   (a) Construct an NFA that accepts the strings in Σ^* where at least one of the last two symbols is an a.

   (b) Convert your NFA to a DFA using the subset construction. Give the DFA both in tabular form and as a transition diagram.

7. Let $\Sigma = \{0, 1\}$. Design ϵ-NFA's for the following languages.

   (a) The set of string that consists of either 01 repeated one or more times or 010 repeated one or more times.

   (b) The set of strings such that at least one of the last ten positions is a 1.