COMP 335 – Introduction to Theoretical Computer Science

Solution to Assignment 1

Q1.1. $\Sigma = \{a, b\}$.
$L_1 = \{w : w = bxyaz \text{ and } x, y, z \in \Sigma^*\}$.

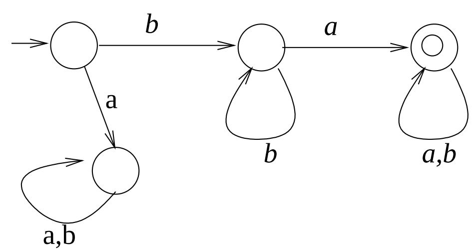

Fig. 1. DFA for L_1 defined in Q1

Q1.2. $\Sigma = \{a, b\}$.
$L_2 = \{w : w \in \Sigma^* \text{ and } |w| = 2k + 1, k \geq 0\}$.
i.e., $|w| = 1, 3, 5, \ldots$ for $k = 0, 1, 2, 3, \ldots$.

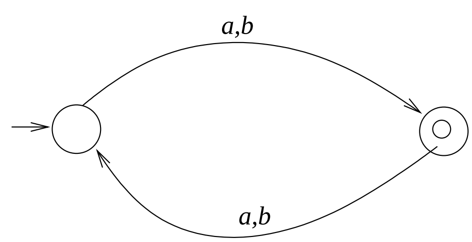

Fig. 2. DFA for L_2 in Q1.

Q1.3. $\Sigma = \{a, b\}$.

$L_3 = \{w : ((n_a(w) + n_b(w))mod3 < 2\}$.

Since á' and b́' are the only symbols in Σ we basically focus on the size of w – there is no requirement regarding the order of a's and b's in w. Note that $|w|$ mod $3 < 2$ means the length of string $|w| = 0, 1, 3, 4, \ldots$, 3k, 3k+1, ...

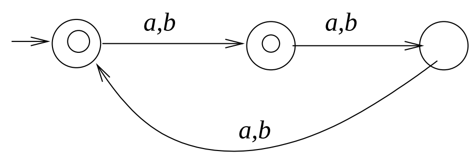

Fig. 3. DFA for L_3 in Q1.

Q1.4. This language includes the strings λ, ba, and every string in $\{a^n b^m : m, n \geq 0\}$ but ab. A DFA for L_4 is shown in Fig. 5.

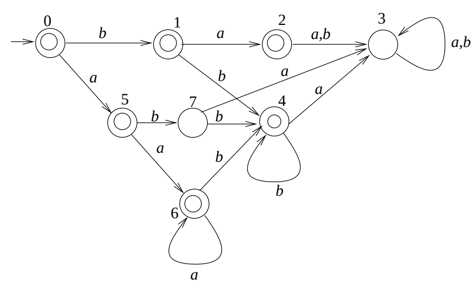

Fig. 4. A DFA for L_4 in Q1

Q2. We use the DFA proposed above for these languages, and transformed them into *generalized transitions diagrams* discussed in the class. Applying the state elimination method repeatedly, we reduce each diagram and finally "read out" the regular expression that they each encode.

For L_1: $r_1 = b(a+b)^*a(a+b)^*$.
For L_2: $r_2 = (a+b)(aa+ab+ba+bb)^*$.
For L_3: $r_3 = ((a+b)(a+b)(a+b))^*(\lambda + a + b)$.

Q3. We first give a DFA M for $L = \{a^n b^m : m, n \geq 0\}$, and then change the status of the states in M from final to non-final and vice versa. This yields the DFA shown in Fig. 4 which accepts $\bar{L}$.

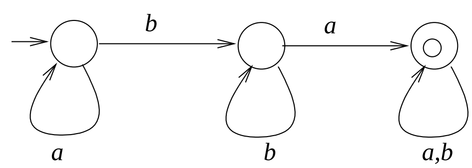

Fig. 5. A DFA for the complement of language L_5 in Q3

Q4. We first draw the transitiin diagram of the given DFA. Using the state elimination technique, we remove all the states (q_2) but the initial state and the only final state. We get a generalized transition diagram which has the following 3 transitions with the corressponding strings and their edges:
The label on the edge from q_0 to q_1 is $0+1$.
The label on the edge from q_1 to q_0 is 0.
The label on the edge from q_1 to q_1 is $1+01$.
A regular expression which can be obtained from the above diagram would be: $r_6 = (0+1)(1+00+01)^*$, where $L(r_6) = L(M)$.

Q5. The minimal DFA is shown in Fig. 6. Steps and details will be discussed in the tutorial.

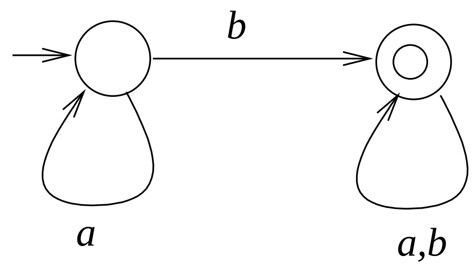

Fig. 6. Minimized DFA