COMP 352/4 **Concordia University** Winter 2001

Data Structures and Algorithms

Mid-term Exam, Sec. Z, Solutions

1. 3 pts. Assume that function $f = 5n^2 + 20n^2 \log n + 15\sqrt{n^5}$ represents the run-time of a program P.

   (a) Consider the 3 terms in f and state which is the most important term, the second most important term, the least important term:
   (1) $15\sqrt{n^5}$, (2) $20n^2 \log n$, (3) $5n^2$.

   (b) Indicate using the big O notation the time complexity of P:
   $O(\sqrt{n^5})$

2. 4 pts. Consider the following function in which only the loops are indicated. All other statements in the function need time $O(1)$.

```
void EX(int n)
 {
   int i = 2*n;
   while (i >= 1){
     ...
     for (int j= 3; j <= n-2; j+=2) {
       ...
      }
     i = i/ 3;
   }
 }
```

   Determine the run-time complexity of EX as a function of n, using the big O notation:

   The while loop is repeated $log_3 2n$ times and the for loop is repeated $(n-4)/2+1$ times. Loops are nested, thus the run-time complexity is $O(nlogn)$.

3. 5 pts.

   (a) Give an informal definition of a queue:
   It is a restricted *list* in which any item is inserted at one end of the list and any deletion is done at the other end of the list.

   (b) Assume we have a circular array implementation of a queue. Draw the picture of the queue and the position of $front$ and $rear$ indices resulting from the following sequence of operations (assume we have a queue of integers):

```
Queue v(6);
v.enqueue(4);
v.enqueue(2);
int i=v.dequeue();
v.enqueue(3);
v.enqueue(i);
```

queue is array[0]...array[6].
queue contains 2 in location 2, 3 in location 3, 4 in location 4.
front=1, rear = 4.

4. 10 pts. Assume that we have a class *list*

```
class list {        // a linked list
private:
 link* head;        // pointer to the list header node
 link* tail;        // pointer to the tail
 link* curr;        // pointer to the current element
public:
  list(const int = LIST_SIZE); //constructor
  ~list();         // destructor
                    //here we have the usual operations
                    //on lists
};
```

This list is implemented using a singly linked list, so the class link is defined as

```
class link {
  public:
     ELEM element;   // ELEM value for this node
     link* next;     // pointer to the next node
     link(const ELEM& elemval, link* nextp=NULL);   //constructor
         { element = elemval; next = nextp}
  ~link() { };       // destructor
}
```

We want to add to the class *list* a function

```
void remEvery(const ELEM & item);
```

which removes, in the linked list pointed to by *head*, every node containing *item*.

```
void remEvery(const ELEM & item){
 if (head==tail) return;          // list is empty
 link * ltemp1 = head;            // start from the head nodde
 link * ltemp2 = ltemp1->next;    // ltemp2 is the node following ltemp1
                                  // in the list
 while (ltemp2 != NULL) {
      if (ltemp2->elem == item)       // delete ltemp2 node
         { ltemp1 ->next = ltemp2->next;
           if (ltemp2 == curr) curr = ltemp1; // node pointed to by
                     // current pointer is deleted, adjust curr pointer
           delete(ltemp2);
          }
      else ltemp1 = ltemp2;    // shift ltemp1 to the next node in the list
      ltemp2 = ltemp1->next    // shift ltemp2 to the next node in the list
    }
 tail = ltemp1;                  // adjust tail pointer
}
```

5. 5 pts.

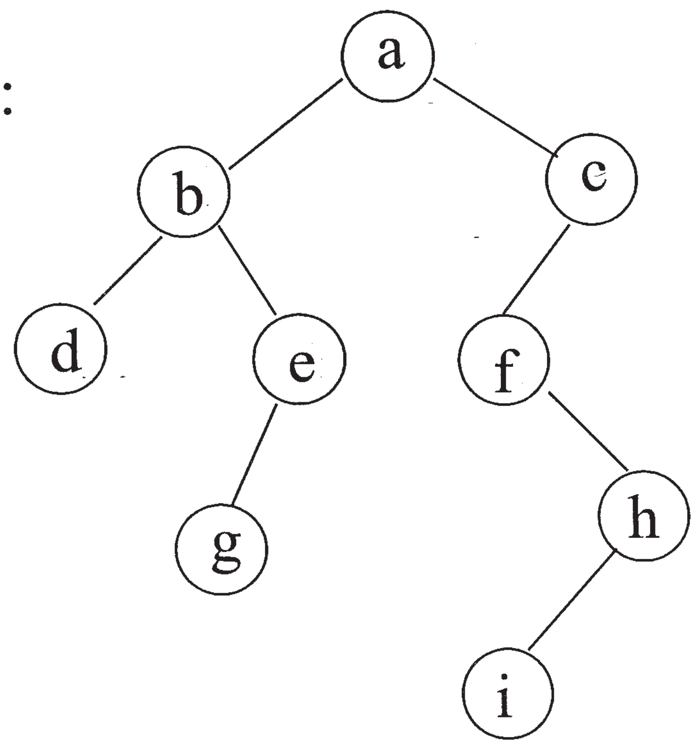
(a) How many internal nodes are there in T?
6

(b) Give the preorder and postorder traversals of the T.
preorder: $a, b, d, e, g, c, f, h, i$
postorder: $d, g, e, b, i, h, f, c, a$

(c) At least how many nodes do you have to add to T to make it a full binary tree?
4

(d) At least how many nodes do you have to add to T to make it a complete binary tree?
17

T:



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CONCORDIA UNIVERSITY
COMP 352 : Data Structures and Algorithms
Fall 2000
Section: V

Midterm Examination

October 17th, 2000
Total Marks: 20 (20% of the final grade)

Name: [redacted]

Student ID: [redacted]

-0.5

Question1 (3 marks)

What is the asymptotic time complexity in the average case for the following operations

(a) Create an array-based stack $\Theta(1)$

(b) Create a linked stack $\Theta(1)$

(c) Find an element in a linked list $\Theta(n)$

(d) Insert an element into a linked queue (enqueue operation) $\Theta(1)$

(e) Insert an element into an array-based queue (enqueue operation) ~~$\Theta(n)$~~ $\Theta(1)$

(f) Clear an array-based list $\Theta(1)$

Question 2 (3 marks)

(a) For the following code fragment give

T(*n*) and Θ(*n*) in the best case $T(n) = c_1$; $\Theta(1)$

T(*n*) and Θ(*n*) in the average case $T(n) = \frac{n(n+1)}{2}$; $\Theta(n)$

T(*n*) and Θ(*n*) in the worst case $T(n) = n$; $\Theta(n)$

```
int SeqSearch (int* array, int k) {          // Find element k
        for (int i=1;i<n; i++)                     // For each element
                if (array[i] = k)            // If found
                        return i;            // Return its position
        return NOT_FOUND;                    // Return const - flag
}
```

(b) For the following code fragment give

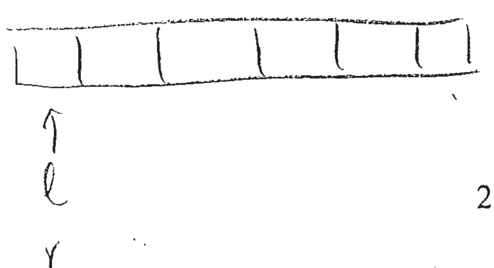
T(*n*) and Θ(*n*) in the best case $T(n) = c_1$; $\Theta(1)$ // c_1 is constant

T(*n*) and Θ(*n*) in the average case $T(n) = \log n$; $\Theta(\log n)$

T(*n*) and Θ(*n*) in the worst case ~~$T(n) = n/2$; $\Theta(n)$~~

−0.5

```
int binarySearch (int* array, int k, int left, int right) {
        // find element k
        int l = left + 1;               //l and r beyond the bounds
        int r = right + 1;
        while (l + 1 != r){
                int i = (l + r)/2
                if ( k == array[i]) return i; // return its position
                else if ( k < array[i]) r = i;      // in left half
                else ( k > array[i]) l = i;   //in right half
        }
        return NOT_FOUND;               // return const - flag
}
```



Question 3

(a) (1 mark) Convert the following infix expression into a postfix form

(5 + 1) * (3 - 2) / 3 - 4 * 2

51+32-*3/42*-

(b) (1 mark) Evaluate the following postfix expression

52+6*93/-

(5+2)*6 - (9/3) = 39

Question 4 (2 marks)

Each data element is 16 bytes. Size of a pointer is 8 bytes. Maximum number of elements is 100. Consider a queue that contains *n* elements. Calculate break-even point beyond which the array based implementation is more space efficient.

P = 8 bytes
E = 16
D = 100

Array = DE
List = n(P+E)

$n > \frac{DE}{P+E}$ ⇒ $n > \frac{1600}{24}$

Thus, by the break-even analysis, it requires at least 15 elements (n≥15) in order for the array-based implementation to be more efficient.

Question 5 (3 marks)

Fill in the code to remove an element from the top of the stack, and return it to the calling routine, in a linked stack implementation with the following definitions:

```
class Node { // Node class
public:
     char aValue;
     Node* next;
};

class Stack { // Linked stack class
private:
     Node* topOfStack;          // Pointer to top element
public:
     Stack() {topOfStack = NULL;}  // Constructor
     Node* Remove ();
     ...
     ...
     ...
};
```

```
Node* Stack::Remove()
{
 Node* temp = topOfStack;   // make "temp" point to sam
 topOfStack = topOfStack->next;  // memory location as topOfSt
                                 // make topOfStack point t
                                 // next node in the list

 return temp;      // return pointer "temp" since
                   // the return argument is
   Empty Stack.    // expecting a pointer
}
```

1

Question 6 (7 marks)

Using the public functions for the class `List` given below, write a C++ function,

```
List*Alternate (List*& L1, List*& L2)
```

that creates a list from elements in L1 and L2 by alternating elements in the two lists and then appending the remaining nodes of the longer of two lists.

For example, if L1 = (10,20, 30) and L2 = (15, 25, 35, 45, 55), the call to `Alternate(L1, L2)` produces the list (10, 15, 20, 25, 30, 35, 45, 55), and L1and L2 become empty lists. Assume that list elements are of type `Elem`.

```
class List { // Linked list class
private:
     Link* head;          // Pointer to list header
     Link* tail;          // Pointer to last Elem
     Link* curr;          // Pos of "current" Elem
public:
     List();                         // Constructor
     ~List();                        // Destructor
     void clear();                   // Remove all Elems
     void insert(const Elem);        // Insert at current pos
     void append(const Elem);        // Insert at tail
     Elem remove();                  // Remove/return Elem
     void setFirst();                // Set curr to first pos
     void prev();                    // Move curr to prev pos
     void next();                    // Move curr to next pos
     int length() const;             // Return length
     void setPos(int);               // Set current pos
     void setValue(const Elem);      // Set current value
     Elem currValue() const;         // Return current value
     bool isEmpty() const;           // TRUE if list is empty
     bool isInList() const;          // TRUE if now in list
     bool find(Elem);                // Find value
};
```

```
while ( L2. isInList( ))
  {
    alter. insert( L1. currValue( ));
    alter. insert( L2. currValue( ));
    L1. remove( );   // remove current pointer of L1
    L2. remove( );   //    "      "      "     " L2
    L1. next( );     //  go to next node in L1
    L2. next( );     //  "  "   "    "    " L2
  } //end of while
  while ( L1. isInList( ))
  {
    alter. append( L1. currValue( ));
    L1. remove( );
    L1. next( );
  } //end of while
} //end of else

} //end of Alternate
```

```
void Alternate (List *& L1, List *&L2) // given
{
  List alter ; // create object of type "List" to make new
                                                // list
  L1->setFirst(); // set pointer to first node in L1
  L2->setFirst(); //  "  "  "  "  L2

if (L1->length() <= L2->length())
  {
    while (L1->isInList())
    {
     alter.insert(L1->currValue());
     alter.insert(L2->currValue());
     L1->remove();  // remove current pointer in L1
     L2->remove();  //  "  "  "  "  L2

     L1.next();   // go to next node
     L2.next();   // go to "  node
    } //end of while
   while(L2.isInList())
    {
     alter.append(L2.currValue());
     L2.remove();
     L2.next();
    } // end of while
  } // end of if
else
  {
   // see next page
```

—2,5

CONCORDIA UNIVERSITY
COMP 352 : Data Structures and Algorithms
Summer 2001

Midterm Examination

May 30th, 2001
Total Marks: 50 (25% of the final grade)

Name:______________________________________

Student ID:__________________________________

Question 1 (9 marks)

Give the asymptotic time complexity in the average case for the following operations:

(a) Clear a linked list ______________________________

(b) Delete an element in an array based list______________________________

(c) Find an element in a linked list ______________________________

(d) Create a linked stack ______________________________

(e) Insert an element into a linked stack (push operation) ______________________

(f) Insert an element into an array-based stack (push operation) ________________

(g) Create a linked queue ______________________________

(h) Insert an element into a linked queue (enqueue operation) __________________

(i) Insert an element into an array-based queue (enqueue operation) _____________

Question 2 (8 marks)

Give T(*n*) and Θ(*n*) for the following code fragments

(a)

```
sum = 0;
for (j=1; j<=n; j++)
  for (i=1; i<=n; i++)
    sum++;
for (k=0; k<n; k++)
  A[k] = k;
```

(b)

```
sum2 = 0;
for (i=1; i<=n; i++)
  for (j=1; j<=i; j++)
    sum2++;
```

(c)

```
sum1 = 0;
for (k=1; k<=n; k*=2)
  for (j=1; j<=k; j++)
    sum1++;
```

(e)

```
sum = 0;
k = 5;
for (i=0; i<k; i++)
  for (j=0; j<n; j++)
    sum++;
```

Question 3 (3 marks)

Each data element is 16 bytes. Size of a pointer is 8 bytes. Maximum number of elements is 100.

(a) Determine the space required for a linked list implementation (Number of elements in the list is *n*)

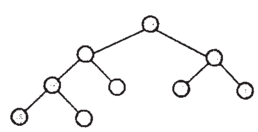
(b) Determine the space required for an array based implementation (Number of elements in the list is *n*)

(c) Calculate break-even point (Break-even point is the number of elements at which both the linked list and array-based list implementations are of equal space efficiency.)

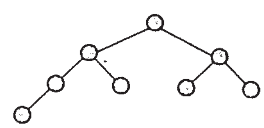
Question 4 (5 marks)

For each of the following trees please state whether it is full, complete, neither, or both

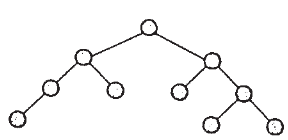
(a)



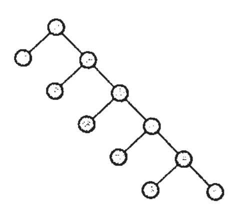
(d)



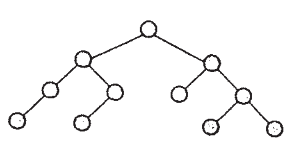
(b)



(e)



(c)



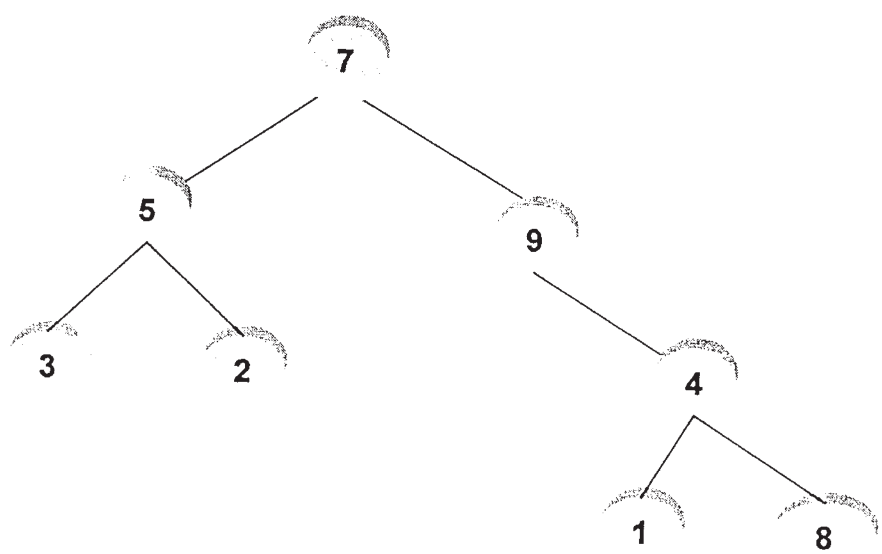
Question 5 (4 marks)

(a) Traverse the tree below using inorder traversal (Write the numbers corresponding to the nodes).

____

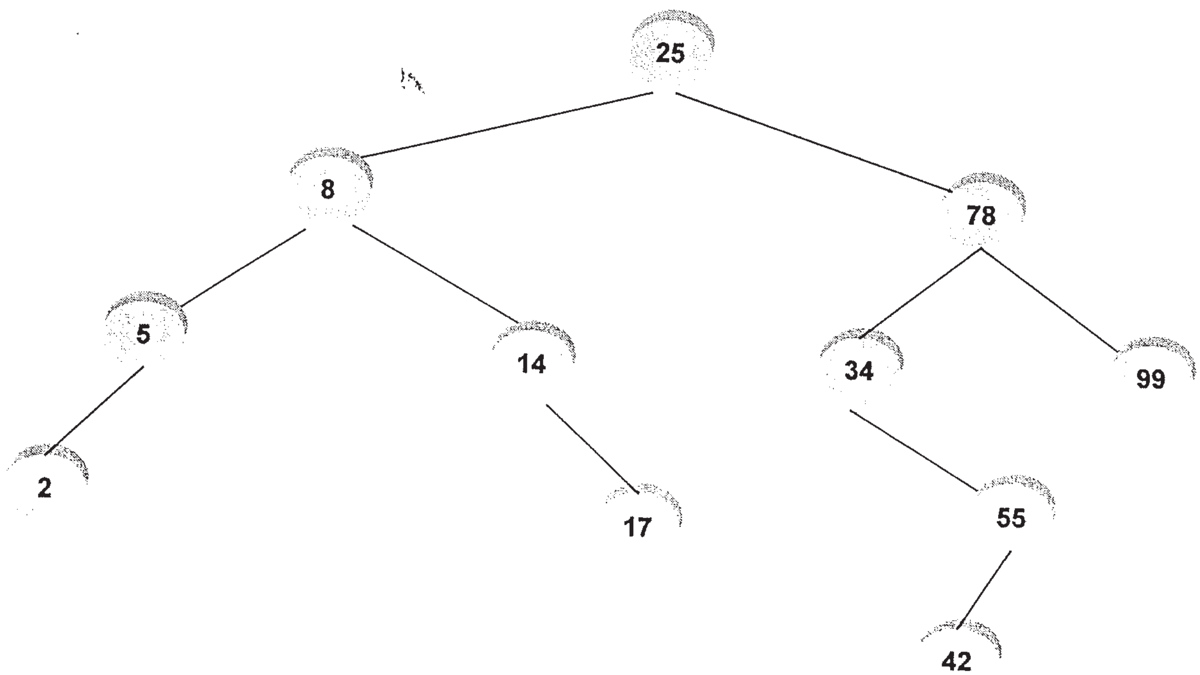
(b) Traverse the tree below using preorder traversal (Write the numbers corresponding to the nodes).

____



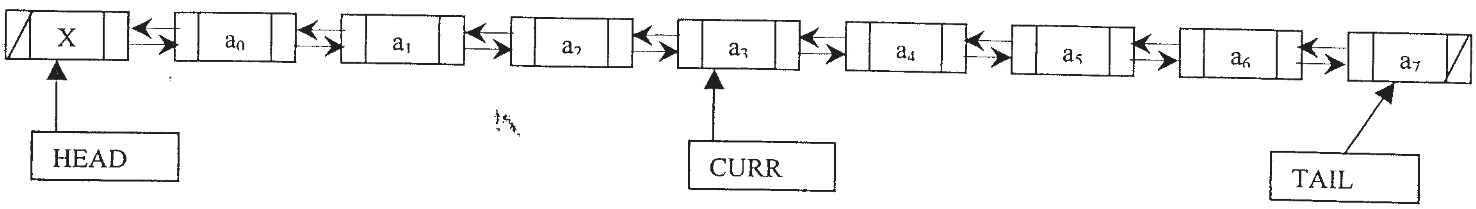
Question 6 (5 marks)

Draw the BST that results from deleting value 25 from the BST bellow

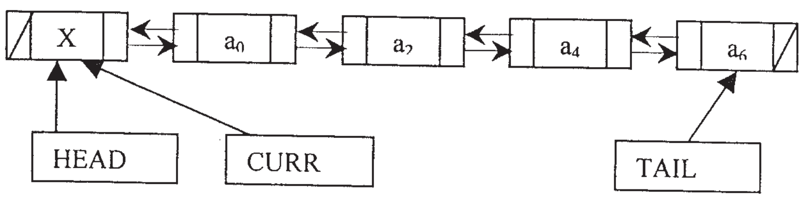


Question 7 (16 marks)

Given a doubly linked list A



the *trim* operation applied to list A gives



that is, all elements a_i with odd subscript i are removed.
Add to the doubly linked list class implementation a member function

```
void trim();
```

which trims the list. To keep this question simple, `trim()` should just reset the `curr` pointer to the head of the list.

Try to make your function as efficient as possible.

You should assume the doubly – linked list node and doubly – linked list classes as given in your textbook. For your reference, the declarations for the list node class and for the linked list class are reproduced below:

```
class Link {                    // Doubly - linked node
public:
      Elem element;             // Elem value for node
      Link *next;               // Pointer to next node
      Link* prev;               // Pointer to prev node
      Link(const Elem elemval, Link* nextval =NULL,
                                     Link* prevp =NULL)
      { element = elemval; next = nextval; prev = prevp;}
      Link(Link* nextval =NULL, Link* prevp = NULL)
      { next = nextval; prev = prevp;}
};

class List { // Linked list class
private:
      Link* head;               // Pointer to list header
      Link* tail;               // Pointer to last Elem
      Link* curr;               // Pos of "current" Elem
public:
      List();                             // Constructor
```

```
    ~List();                              // Destructor
    void clear();                         // Remove all Elems
    void insert(const Elem);              // Insert at current pos
    void append(const Elem);              // Insert at tail
    Elem remove();                        // Remove/return Elem
    void setFirst();                      // Set curr to first pos
    void prev();                          // Move curr to prev pos
    void next();                          // Move curr to next pos
    int length() const;                   // Return length
    void setPos(int);                     // Set current pos
    void setValue(const Elem);            // Set current value
    Elem currValue() const;               // Return current value
    bool isEmpty() const;                 // TRUE if list is empty
    bool isInList() const;                // TRUE if now in list
    bool find(Elem);                      // Find value
};
```